The Mittag-Leffler theorem for proper minimal surfaces and directed meromorphic curves

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Abstract Given an open Riemann surface M and a closed conical complex subvariety $A \subset \mathbb{C}^n$ $(n \geq 3)$, a holomorphic map $F : M \to \mathbb{C}^n$ is an *A-immersion* if its complex derivative F' on M with respect to any local holomorphic coordinate on M assumes values in $A_* = A \setminus \{0\}$. Given a closed discrete subset $E \subset M$, by a *meromorphic A-immersion* $M \setminus E \to \mathbb{C}^n$ we mean a holomorphic A-immersion $M \setminus E \to \mathbb{C}^n$ extending to M as a meromorphic map with effective poles at all points in E.
We aim to connect this notion to a well-known result from complex analysis, the Mittag-Leffler theorem from 1884, which states that for any closed discrete subset $E \subset \mathbb{C}^n$ and a meromorphic function f on a neighbourhood of E there exists a meromorphic function g on $\mathbb{C}$ which is holomorphic on $\mathbb{C} \setminus E$ and the difference $g - f$ is holomorphic at every point of E. This was extended by H. Florack in 1948 to functions on open Riemann surfaces. In 2022, A. Alarcón and F. J. López in [2] proved an analogue of the Mittag-Leffler theorem for complete conformal minimal surfaces in $\mathbb{R}^n$, including approximation and interpolation (see also [3, Chapter 3]). We generalize the latter to obtain a Mittag-Leffler-type theorem for proper directed immersions $M \to \mathbb{C}^n$ on any open Riemann surface M and state some consequences describing properties of minimal surfaces in $\mathbb{R}^3$. This is joint work with Antonio Alarcón.

References

[1] A. Alarcón and F. Forstnerič. Null curves and directed immersions of open Riemann surfaces. *Invent. Math.*, 196(3):733–771, 2014.

[2] A. Alarcón and F. J. López. Algebraic approximation and the Mittag-Leffler theorem for minimal surfaces. *Anal. PDE*, 15(3):859–890, 2022.

[3] A. Alarcón, F. Forstnerič, and F. J. López. *Minimal surfaces from a complex analytic viewpoint.* Springer Monogr. Math. Cham: Springer, 2021.

[4] Y. Kusunoki and Y. Sainouchi. Holomorphic differentials on open Riemann surfaces. *J. Math. Kyoto Univ.*, 11:181–194, 1971.

[5] F. J. López. Exotic minimal surfaces. *J. Geom. Anal.*, 24(2):988–1006, 2014.