Periodic motions in Electrodynamics: the Infinite Wire model

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The motion of a charged particle in an electromagnetic field is modelled by the classical Lorentz Force equation

$$\frac{d}{dt}\left(\frac{\dot{q}(t)}{\sqrt{1-|\dot{q}(t)|^2}}\right) = E(t,q(t)) + \dot{q}(t) \times B(t,q(t)), \tag{1}$$

where $q : [0,T] \to \mathbb{R}^3$ denotes the position of the particle and $(E,B) : [0,T]\mathbb{R}^3 \to \mathbb{R}^6$ is the electromagnetic vector field. On the right side we have the common expression of the Lorentz Force, while the left is the relativistic acceleration of the particle, that implies the Special Relativity limitation for the velocity $\dot{q}(t)$.
In this talk we shall focus on the electromagnetic regime induced by a periodically time dependent current $I_0 + kI(t)$ along an infinitely long and infinitely thin straight wire, where $I_0 > 0$ and $k \geq 0$ are constants and $I(t)$ is a T-periodic function. In that sense, the electromagnetic field is obtained by solving the Maxwell's equations with the corresponding current distribution $\vec{J}$ as data. Due to the cylindrical symmetries, there are conserved quantities for any $k \geq 0$ that reduce the dynamics in (1) to a scalar equation for the radial variable (in cylindrical coordinates). By using global continuation and topological degree, we identify a bi-parametric family of radially periodic solutions for (1). The proofs involve some delicate estimations of the induced electromagnetic field (as solution of Maxwell's equations), which can be of independent interest.

References

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[2] M. Garzón, S. Marò: *Motions of a charged particle in the electromagnetic field induced by a non-stationary current.* Physica D: Nonlinear Phenomena, 2021, vol. 424, p. 132945.
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