Title: A characteristic mapping method for nonlinear transport equations: applications to incompressible Euler, MHD and Vlasov-Poisson

Abstract:

We present an efficient semi-Lagrangian Characteristic Mapping (CM) method for solving nonlinear transport equations. This method evolves advected quantities by discretizing the flow map associated with the velocity field. Using the properties of the Lie group of volume preserving diffeomorphisms SDiff, long-time deformations are computed from a composition of short-time submaps which can be accurately evolved on coarse grids. We demonstrate through numerical experiments the validity of the method and show that energy is not dissipated through artificial viscosity and small scales of the solution are preserved. We provide error estimates and numerical convergence tests showing that the method is globally third-order accurate. Applications are presented for the incompressible Euler equations in 2d and 3d, magnetohydrodynamics in 2d and the Vlasov-Poisson system 1d-1d. This is joint work with Xi-Yuan Yin, Philipp Krah, Julius Bergmann and Jean-Christophe Nave.

References:

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