MULTIPLE ORTHOGONAL POLYNOMIALS. INTRODUCTION, APLICATIONS AND EXTENSION TO THE BIVARIATE CASE

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Abstract

Orthogonal Polynomials Theory is a brach of Approximation Theory where, providing the vector space of polynomials $\mathbb{R}[x]$ with an integral inner product which depends on a real measure μ, it is possible to obtain basis of $\mathbb{R}[x]$ with several properties and applications to other areas: differential equations, approximation of functions, interpolation,...

Moreover, there exist some generalizations of the univariate orthogonal polynomials theory to the multivariate case. This is, orthogonal polynomials in the vector space of multivariate polynomials $\mathbb{R}[x_1, \ldots, x_d]$.

Furthermore, another extension of standard Orthogonal Polynomials are those known as Multiple Orthogonal Polynomials (MOPs), which satisfy orthogonality conditions concerning multiple measures $\mu_1, \ldots, \mu_r$, and play significant role in several applications such as Hermite-Padé approximation, random matrix theory or integrable systems. However, this theory has only been studied in the univariate case. In this poster, some generalized definitions of the two main types of multiple orthogonality are given, together with some examples and extended results.

Keywords: Orthogonal Polynomials, Approximation Theory, Applications, Multiple orthogonality.
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