Title: Exploring global dynamics of nonlinear PDEs with validated numerics

Abstract:

Solutions to nonlinear PDEs are notoriously difficult to obtain and are often only accessible through numerical methods. While abstract theorems can tell us about the convergence rate of a numerical method (for example, in the limit as the spatial discretization and the time step go to zero), this typically does not offer an a posteriori error estimate: That is, how close is the specific solution I found on my computer to the true result?

In this talk, I will discuss a complex-scalar PDE which may be seen as a toy model for vortex stretching in fluid flow, and cannot be neatly categorized as conservative nor dissipative. In a recent series of papers, we have shown that this equation exhibits rich dynamical behavior existing globally in time: non-trivial equilibria, stable/unstable manifolds, homoclinic orbits, and heteroclinic orbits. By utilizing validated numerics to synthesize all the forms of error in a calculation, we are able to obtain computer-assisted-proofs that our numerical solution is quantifiably close to the true solution.