

"Let X be a smooth projective variety of dimension n over an algebraically closed field K of characteristic 0, and let $M_H = M(X, H)(r; c_1, \dots, c_s)$ be the moduli space of rank- r vector bundles on X with respect to an ample divisor H on X and with fixed Chern classes $c_i(E) = c_i$ for $i=1, \dots, s := \min\{r, n\}$. One way to study the geometry of these moduli spaces is by examining their subvarieties. Among them one can try to study Brill-Noether loci $W_H^k(r; c_1, \dots, c_s)$, whose points correspond to stable vector bundles having at least k independent sections. In my talk, after introducing some background material, I will introduce these subvarieties and I will present some questions related to them. Finally I will focus my attention on new results concerning the non-emptiness of the Brill-Noether locus in the case of rank-2 stable vector bundles on ruled surfaces."