Asymptotic behaviour of solutions to the Becker-Döring equations

Coagulation and fragmentation equations constitute mathematical models which describe how particles aggregate (coagulation) and divide (fragmentation) within a population. These equations, used in fields such as chemistry, physics, and biology, play a crucial role in understanding the dynamics of particle systems. Coagulation equations illustrate how particles merge to form larger entities, while fragmentation equations depict the breakdown of larger particles into smaller ones.

The Becker-Döring equations, introduced by Becker & Döring (1935), serve as fundamental frameworks for describing the first-order phase transitions in various chemical and physical systems, such as vapor condensation and lipid aggregation. Over the years, considerable attention has been devoted to studying this model in the nonlinear case in contrast to the linear situation. For this reason, we focus on the linear Becker-Doring equations, defined by:

$$\frac{d}{dt}c_i(t) = W_{i-1} - W_i, \quad \text{for all } i \geq 2$$

and c_1 is constant for all $t \in \mathbb{R}$, where the fluxes W_i depends on the coagulation and fragmentation coefficients a_i and b_i, respectively, by the expression $W_i = a_i c_1 c_i - b_{i+1} c_{i+1}$.

Our aim is to examine the long-term behavior of solutions to the equilibria, providing a more accurate estimation of the convergence velocity than the shown by Kreer (1993). To achieve this, we employ spectral theory and entropy methods using the free energy provided also by Kreer (1993). We interpret the metastability of the equations in terms of some of our results.