Title: Invariant manifolds in the incompressible Euler equation

Abstract: I will review recent results on existence and dynamics of finite dimensional invariant manifolds of the Euler equations of hydrodynamics. These are families of velocity fields, parametrized by some parameter space N, with the property that the solutions of the Euler equation with initial condition in the family exist and remain there for all time, defining a finite-dimensional ODE on the parameters. In particular, I will show that invariant manifolds with many types of interesting dynamics, like quasiperiodicity and chaos, are present in the Euler equations on certain Riemannian manifolds; in fact, any structurally stable dynamics can appear if the fluid domain is high dimensional enough. Finally, some open problems suggested by these results will be presented.