Overdetermined problems in the sphere and minimal surfaces in the Euclidean ball

José M. Espinar[†] and Diego A. Marín[‡]

[†] Department of Geometry and Topology and Institute of Mathematics (IMAG), University of Granada, 18071, Granada, Spain; e-mail: jespinar@ugr.es

[‡] Department of Geometry and Topology and Institute of Mathematics (IMAG), University of Granada, 18071, Granada, Spain; e-mail: damarin@ugr.es

Abstract

Let $\Omega \subset \mathbb{S}^2$ be a domain with $\mathcal{C}^2$-boundary and $\xi \in \mathcal{C}^2(\Omega)$. An overdetermined elliptic problem (O.E.P.) is an elliptic partial differential equation in which more than one boundary data is imposed. In this talk we study the following O.E.P.,

$$\begin{cases} \Delta^{\mathbb{S}^2}\xi + 2\xi = 0 & \text{in} \quad \Omega, \\ \xi = 0 & \text{along} \quad \partial\Omega, \\ |\nabla^{\mathbb{S}^2}\xi|^2 = b_i^2 & \text{along} \quad \Gamma_i \in \pi_0(\partial\Omega),\ i \in \{1, \dots, k\}, \end{cases}$$

where b_i is a positive constant for each i.

We will see that this problem is related to the theory of minimal surfaces in Euclidean space. In particular, we classify solutions (Ω, ξ) to the previous problem, provided that the function ξ satisfies certain hypothesis, and this gives us a classification result about minimal surfaces inside the Euclidean unit ball. As a corollary of this result, we obtain a new characterization of the critical catenoid among the embedded free boundary minimal annuli in the unit ball. This is based on a joint work with José M. Espinar.