Title: Dual pairs involving the group of volume preserving diffeomorphisms (and coadjoint orbits)

Abstract:

The ideal fluid dual pair due to Marsden and Weinstein [2] consists of the momentum maps on the manifold of embeddings Emb(S, M), for commuting actions of the volume preserving diffeomorphism group of S and the Hamiltonian group of M. Holm and Marsden [1] observed that commuting Hamiltonian actions of Diff(M) and Diff(S) on the cotangent bundle of Emb(S, M) also yield a dual pair. In a recent preprint [3] we show that it remains a dual pair when Diff(M) is replaced by its subgroup of volume preserving diffeomorphisms.

Via symplectic reduction we obtain descriptions of coadjoint orbits of the group of volume preserving diffeomorphisms in terms of nonlinear Grassmannians of decorated/augmented submanifolds. We give applications to singular vortex configurations for the incompressible Euler equations in 2D and 3D.

[1] D.D. Holm and J.E. Marsden, Momentum maps and measure-valued solutions (peakons, filaments and sheets) for the EPDiff equation, In The Breadth of Symplectic and Poisson Geometry, 203-235, Progr. Math., 2004

[2] J. E. Marsden and A. Weinstein, Coadjoint orbits, vortices, and Clebsch variables for incompressible fluids. Phys. D 7(1983), 305-323

[3] S. Haller and C. Vizman, A dual pair for the group of volume preserving diffeomorphisms, arXiv:2405.10737, 2024

This is joint work with Stefan Haller from the University of Vienna.