Asymptotic estimates for the Dirichlet heat equation in exterior domains

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Abstract

We give large-time L^1 and uniform sharp asymptotic estimates for solutions of the Dirichlet heat equation in the complement of a bounded open set of $\mathbb{R}^d$ with certain technical assumptions. We show that the heat kernel behaves asymptotically as the heat kernel in the full space, with a factor that takes into account the shape of the domain through the *réduite* function, and a second factor which accounts for the loss of mass through the boundary. This improves for large times the well-known estimates from **?** and **?**. All estimates include an explicit rate of approach to the asymptotic profiles at the different scales natural to the problem, and are also given for general solutions of the heat equation in exterior domains for initial data with suitable finite moments. The estimates on the rate of approach to the asymptotic profiles are new, even when the domain is the complement of the unit ball in $\mathbb{R}^d$.

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