Consider a smooth bounded domain $\Omega \subset \mathbb{R}^d$, $d \geq 2$, and the eigenvalue problem supplemented with boundary conditions

$$\begin{aligned} &\Delta u + \lambda u = 0 \quad \text{in } \Omega\,, \\ &\cos\sigma\, \partial_\nu u + \sin\sigma\, u = 0 \quad \text{on } \partial\Omega \end{aligned} \tag{1}$$

for some fixed constant σ.

A simple blowup argument shows that the local behavior of a high energy eigenfunction over length scales of order $\lambda^{-\frac{1}{2}}$ must be described by a monochromatic wave, that is, a solution to the Helmholtz equation

$$\Delta\varphi + \varphi = 0\ ,\ \text{on } \mathbb{R}^d. \tag{2}$$

Moreover, given any point $x \in \Omega$ and a monochromatic wave φ, there is an approximate eigenfunction (that is, a linear combination of eigenfunctions with $\lambda_n \in [\lambda, (1+\delta)\lambda]$ with any fixed $\delta \ll 1$ and sufficiently large $\lambda \gg 1$) of energy λ which looks like φ over balls centered at x and of radius $\lambda^{-\frac{1}{2}}$. This fact is independent of the geometry of the domain Ω.

A central question in spectral geometry is how the orbit structure of the corresponding classical system (which in this case is the dynamical billiard defined by the planar region Ω) affects this connection between high energy eigenfunctions and monochromatic waves. In this talk, we address that question in the case of polygonal domains with integrable dynamical billiard.