Topology of real quadratic mappings into the plane

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For a pair of multivariate real quadratic forms, we consider a natural counterpart of the numerical range of complex square matrix and present several related topological results.

The main attention is given to the singularities of the associated mapping from a real projective space into the plane. For a generic pair of real quadratic forms with an odd number of variables, we prove that the discriminant curve of the aforementioned mapping always contains an odd number of cusp points.

For a pair of quadrivariate real quadratic forms, we compute the bifurcation diagram of the numerical range and calculate Euler characteristics of its fibres using the so-called signature formulas for topological invariants developed in our previous papers. Combining these results with the topological classification of intersections of two real quadrics obtained by S.Lopez de Medrano, we become able to give a complete list of the possible topological types of the fibres of generalized numerical range in four dimensions. Under the same assumptions we also list all possible types of isolated singular points of the fibres of generalized numerical range. More detailed results are obtained for pairs of binary and ternary quadratic forms using the case-by-case analysis of the normal forms of quadratic mappings into the plane given in recent papers of M. Farnik and Z. Jelonek.