Extendable t-structure and finitistic dimension of small triangulated categories

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(joint work with R. Biswas, H. X. Chen, K. M. Rahul, and C. J. Parker)

Abstract

A **good metric** $\mathcal{M} = \{\mathcal{M}_n\}_{\mathbb{N}}$ on a triangulated category $(\mathcal{S}, \Sigma)$ is a sequence of additive extension-closed subcategories such that $\Sigma^i \mathcal{M}_{n+1} \subseteq \mathcal{M}_n \subseteq \mathcal{S}$, for all $n \in \mathbb{N}$ and $i \in \{-1, 0, 1\}$. For $\mathcal{S}$ small, Neeman has recently constructed, for any good metric $\mathcal{M}$ on $\mathcal{S}$, a new small triangulated category $\mathfrak{S}_{\mathcal{M}}(\mathcal{S})$, called the **$\mathcal{M}$-completion** of $\mathcal{S}$, as a full subcategory of Mod-$\mathcal{S} := [\mathcal{S}^{\mathrm{op}}, \mathrm{Ab}]$.

After extending the assignment $\mathcal{S} \mapsto \mathfrak{S}_{\mathcal{M}}(\mathcal{S})$ to a map sending each subcategory $\mathcal{X} \subseteq \mathcal{S}$ to a suitable $\mathfrak{S}_{\mathcal{M}}(\mathcal{X}) \subseteq \mathfrak{S}_{\mathcal{M}}(\mathcal{S})$, we isolate a class of t-structures in $\mathfrak{S}$, called **$\mathcal{M}$-extendable**, via a natural compatibility condition with $\mathcal{M}$. For any $\mathcal{M}$-extendable t-structure $\mathfrak{t} = (\mathcal{D}^{\leqslant 0}, \mathcal{D}^{\geqslant 1})$ in $\mathcal{S}$, one gets a new t-structure $\mathfrak{S}_{\mathcal{M}}(\mathfrak{t}) := (\mathfrak{S}_{\mathcal{M}}(\mathcal{D}^{\leqslant 0}), \mathfrak{S}_{\mathcal{M}}(\mathcal{D}^{\geqslant 1}))$ in $\mathfrak{S}_{\mathcal{M}}(\mathcal{S})$, whose heart is equivalent to that of $\mathfrak{t}$. Moreover, $\mathfrak{S}_{\mathcal{M}}(\mathfrak{t})$ is bounded above, if so is $\mathfrak{t}$.

In the second part of the talk, after recalling a recent construction of Neeman that associates to any object $G \in \mathcal{S}$ a suitable good metric $\mathcal{M}_G$, we will discuss a new notion of (local) **finitistic dimension** $\mathrm{fin.dim}(\mathcal{T}, H)$ of any small triangulated category $\mathcal{T}$ at one of its objects $H \in \mathcal{T}$, including a comparison with previous notions of dimension in the triangulated context. An important feature of this new invariant is the following: for any object G in a small triangulated category $\mathcal{S}$, the condition $\mathrm{fin.dim}(\mathcal{S}^{\mathrm{op}}, G) < \infty$ forces all bounded t-structure in $\mathcal{S}$ to be $\mathcal{M}_G$-extendable. As a consequence, we will verify that the condition $\mathtt{fin}.\mathrm{dim}(\mathcal{S}^{\mathrm{op}}, G) < \infty$ (for some $G \in \mathcal{S}$) forces the following dichotomy: either in $\mathcal{S}$ there are no bounded t-structures at all or, when at least one such t-structure exits, all the others have to be equivalent to (i.e., at finite distance from) it.

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