

A gentle introduction to l^2 -Betti numbers and its applications

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Abstract. Classical Betti numbers are useful homotopy invariants associated with a finite CW-complex. These invariants, however, do not take into account the action of its fundamental group on its universal cover. l^2 -Betti numbers arise from this observation as an equivariant version of their older cousins, the Betti numbers. They were introduced in the 70's by M. Atiyah, while working towards an extension of its Atiyah-Singer Index Theorem to the non-compact setting.

Atiyah himself observed that, in case of having a finite fundamental group, all these new invariants coincide with the previous Betti numbers, properly normalized by the size of the group. In particular, they are all rational numbers. This fact resulted in Atiyah wondering if this was true not only for finite fundamental groups, but for countably infinite ones.

In this talk, I plan to give a gentle introduction to these l^2 -invariants. I will present several applications of them in relation to problems in many branches of mathematics, as well as important conjectures that were directly derived from Atiyah's original question.

I will also present some results concerning the explicit computation of l^2 -Betti numbers, showing their irrationality (first proved by Austin in 2013) in particular examples.