TORSION PAIRS VIA THE ZIEGLER SPECTRUM

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Abstract. The torsion pairs in the category $\mathrm{mod}(A)$ of finite dimensional modules over a finite dimensional algebra A form a complete lattice $\mathbf{tors}(A)$ which encodes essential information on A. Another important measure for the complexity of the category $\mathrm{mod}(A)$ is given by the set $\mathbf{brick}(A)$ of isomorphism classes of finite dimensional bricks, i.e. modules whose endomorphism ring is a skew-field.

The seminal work of Adachi, Iyama and Reiten and of Demonet, Iyama and Jasso studies $\mathbf{tors}(A)$ and $\mathbf{brick}(A)$ in terms of silting theory. But unfortunately only torsion pairs and bricks satisfying certain finiteness conditions are captured in this way.

The aim of my talk is to lift these restrictions and describe the whole lattice $\mathbf{tors}(A)$ and the entire collection $\mathbf{brick}(A)$. To this end it is convenient to work with the dual concept of a cosilting complex. Since cosilting complexes are pure-injective, we then have access to the techniques afforded by the theory of purity and can employ the Ziegler spectrum $\mathbf{Zg}(A)$, a topological space associated to A. We will see that the lattice $\mathbf{tors}(A)$ is isomorphic to a lattice determined by closed sets in $\mathbf{Zg}(A)$, and there is a one-one correspondence between the collection $\mathbf{brick}(A)$ and certain points in $\mathbf{Zg}(A)$.

This is a report on joint work with Rosanna Laking and Francesco Sentieri.