Composition of analytic paraproducts

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Let $\mathcal{H}(\mathbb{D})$ denote the space of analytic functions on the unit disc $\mathbb{D}$ of the complex plane. For a fixed analytic function g on the unit disc $\mathbb{D}$, we consider the analytic paraproducts induced by g, which are defined by $T_g f(z) = \int_0^z f(\zeta) g'(\zeta)\, d\zeta$, $S_g f(z) = \int_0^z f'(\zeta) g(\zeta)\, d\zeta$, and $M_g f(z) = f(z)g(z)$. The boundedness of these operators on various spaces of analytic functions on $\mathbb{D}$ is well understood. The original motivation for this work is to understand the boundedness of compositions of two of these operators, for example T_g^2, $T_g S_g$, $M_g T_g$, etc. Our methods yield a characterization of the boundedness of a large class of operators contained in the algebra generated by these analytic paraproducts acting on the classical weighted Bergman and Hardy spaces in terms of the symbol g. In some cases it turns out that this property is not affected by cancellation, while in others it requires stronger and more subtle restrictions on the oscillation of the symbol g than the case of a single paraproduct. In particular we show that the boundedness of a N-letter g-word on A^p_α only depends on the symbol g, N and the total number of T_g's that it contains. Here an N-letter g-word is the composition $L = L_1 \cdots L_N$ of N operators L_j, where each L_j is either of the analytic paraproducts T_g S_g or M_g.

These results are part of of two joint works together with A. Aleman, Carmé Cascante, Joan Fábrega and Daniel Pascuas.

References

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