

Large-scale brain oscillations may reflect the synchronous behavior of neuron populations. However, the mechanisms underlying collective neuron dynamics are not well understood. One approach is to model neuron populations as systems of oscillators: ordinary differential equations with stable limit-cycle solutions. Arnold tongues characterize synchronization regimes of coupled oscillators as a function of frequency difference and coupling strength. However, analysis is often complicated by high dimensionality and nonlinear dynamics.

Phase reduction is effective for studying systems of coupled deterministic oscillators by describing the synchronized dynamics in terms of a one-dimensional phase (timing) variable for each oscillator. However, neural activity is noisy; *stochastic* phase concepts can extend phase reduction to noisy oscillators. Of particular utility is the asymptotic stochastic phase, derived from the Q function: the slowest decaying mode of the stochastic Koopman operator. The Q function simplifies system dynamics for single oscillators [Pérez et al 2023 PNAS], yet a characterization of the synchronization of *coupled* stochastic oscillators remains an open question.

Here, we compute the Q function of coupled stochastic oscillators for three qualitatively different systems: a two-dimensional ring model, a four-dimensional linear system for which the Q function and low-lying spectrum are known analytically, and a four-dimensional nonlinear system. We demonstrate that in each case the eigenvalues corresponding to the Q function exhibit a qualitatively similar bifurcation, and that the synchronization boundary as a function of frequency difference and coupling strength resembles an Arnold tongue. We argue that our stochastic phase-based approach can contribute to the modeling and analysis of large-scale electrophysiological recordings.