On the Lawson-Osserman conjecture on the minimal surfaces system
joint work with Connor Mooney and Riccardo Tione

In the renowned paper by Lawson and Osserman, *non-existence, non-uniqueness, and irregularity of solutions to the minimal surface system*, Conjecture 2.1 stands out:

Conjecture 2.1: The systems (2.2) and (2.3) are equivalent for any locally Lipschitz function f on Ω.

Here (2.2) is the full minimal surface system

$$\begin{cases} \displaystyle\sum_{i=1}^{n} \frac{\partial}{\partial x^i}(\sqrt{g}g^{ij}) = 0; \quad j = 1, \dots, n \\ \displaystyle\sum_{i,j=1}^{n} \frac{\partial}{\partial x^i}(\sqrt{g}g^{ij}\frac{\partial f}{\partial x^j} = 0, \end{cases}$$

while (2.3) involves only the outer variations:

$$\begin{cases} \displaystyle\sum_{i,j=1}^{n} \frac{\partial}{\partial x^i}(\sqrt{g}g^{ij}\frac{\partial f}{\partial x^j} = 0\,. \end{cases}$$

We affirmatively resolve the conjecture in dimension two. Our main result can be succinctly stated as follows:

Theorem Let $f\colon B_1 \subset \mathbb{R}^2 \to \mathbb{R}^n$ be a Lipschitz critical point of the area functional concerning outer variations, then f is smooth.

Having presented the conjecture and our result, the remainder of the talk will be devoted to outlining the ideas behind the proof and elucidating the role of working in two dimensions.

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In the renowned paper by Lawson and Osserman, *non-existence, non-uniqueness, and irregularity of solutions to the minimal surface system*, Conjecture 2.1 stands out and states roughly:

"The outer variations for the minimal surfaces system is sufficient for a graph that is locally Lipschitz continuous."

put differently "Does the outer variations for a locally Lipschitz continuous graph imply that the inner variation holds as well?"

We affirmatively resolve the conjecture in dimension two. Our main result can be succinctly stated as follows: A two-dimensional graph that is locally Lipschitz continuous and is a critical point of the area with respect to outer variations is smooth.

Having presented the conjecture and our result, the remainder of the talk will be devoted to outlining the ideas behind the proof and elucidating the role of working in two dimensions.