Title. Nonautonomous scalar differential equations: critical transitions, rate-induced tracking and early warning signals

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Abstract. In this talk we will discuss mathematical models of critical transitions in nonautonomous scalar differential equations describing the evolution of a single-species population subject to Allee effect. Critical transitions are large and often irreversible changes in the state of a given system in response to small and slow changes in the input. In particular, we study equations given by a population growth rate that has concave derivative with respect to the population size, which is the situation in several models including the Allee effect. Starting from the formulation of [1], in which the equation under study is a time-dependent transition connecting a system in the past to a system in the future, we incorporate intrinsic time dependence in the past and future equations. The intrinsically nonautonomous nature of the model allows for the inclusion of phenomena ranging from Earth's rotation to the alternation of seasons, which may have an impact on the laws of evolution of the species. The dynamical possibilities of the transition equation are only three: one of them corresponds to the persistence of the species, another to extinction and the third is an unstable case that separates the other two. The dynamical phenomenon behind the described critical transitions, which are the change from one of the stable dynamical cases to the other stable case through some parametric variation, is a saddle-node bifurcation of hyperbolic solutions. The parameter describes possible mechanisms that cause critical transitions: the transition rate, the initial phase of the system, the size of the transition... Special attention is paid to rate-induced tracking, also called overshooting in the literature: a phenomenon in which high transition rates are beneficial to the system under study. Examples of the effectiveness of finite-time Lyapunov exponents as early warning signals of critical transitions in such systems are shown. This is joint work with I.P. Longo, C. Núñez and R. Obaya.

References

[1] P. Ashwin, C. Perryman, S. Wieczorek. *Parameter shifts for nonautonomous systems in low dimension: bifurcation- and rate-induced tipping.* Nonlinearity **30** (6), 2185–2210 (2017).

[2] J. Dueñas, I.P. Longo, R. Obaya. Rate-induced tracking for concave or d-concave transitions in a time-dependent environment with application in ecology. Chaos **33**, 123113 (2023).

[3] J. Dueñas, C. Núñez, R. Obaya. Critical transitions in d-concave nonautonomous scalar ordinary differential equations appearing in population dynamics. *SIAM J. Appl. Dyn. Syst.* **22** (4), 2649-2692 (2023).

[4] J. Dueñas, C. Núñez, R. Obaya. Critical transitions for asymptotically concave or d-concave nonautonomous differential equations with applications in ecology. Preprint, arXiv:2311.17566.