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The Hausdorff dimension of planar elliptic measures

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In 1988 Jones and Wolff showed that, for any planar domain $\Omega \subset \mathbb{R}^2$, the Hausdorff dimension of its harmonic measure ω_Ω is at most 1. This result was improved by Wolff in 1993 by finding a subset $F \subset \partial\Omega$ with σ-finite length and with full harmonic measure $\omega_\Omega(F) = 1$.

In the same direction for higher dimensions, $\mathbb{R}^{n+1}$ with $n \geq 2$, Bourgain in 1987 proved that there exists a constant $b_{n+1} > 0$ such that $\dim_{\mathcal{H}} \omega_\Omega \leq n + 1 - b_{n+1}$. In contrast to what happens in the plane, Wolff in 1995 showed that b_{n+1} can not equal 1 by constructing a set $\Omega_n \subset \mathbb{R}^{n+1}$ satisfying $\dim_{\mathcal{H}} \omega_{\Omega_n} > n$.

We focus on the study of the dimension of planar elliptic measures arising from the PDE $\operatorname{div}(A\nabla\cdot) = 0$ with uniformly elliptic matrix A. In this scenario, we present the analogous result of Jones-Wolff or Wolff in the plane for these situations:

- Reifenberg flat domains with small constant and Lipschitz matrices; and
- Either the matrix is symmetric with determinant 1, or the domain is CDC, via the application of quasiconformal mappings.

This is a joint work with Martí Prats and Xavier Tolsa.