# Improving optimal control in systems with biologically realistic multiplicative and internal noise

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To act in the world, we integrate sensory information as we move our sensors and body through the external environment, creating perception-action loops. Movements alter sensory inputs, which are then compared with predictions and used in planning future movements to accomplish internal objectives. However, these mechanisms are challenged by different noise-sources, coming from the integration of sensory feedback and the motor output itself [1]. Furthermore, neural representations are subject to internal fluctuations, which affect estimation processes and, consequently, behaviour [1, 2]. Stochastic optimal control theory formalizes these concepts to explain behaviour through optimality principles at the algorithmic level [3]. In this context, having an optimal solution is crucial for assessing the rationality of the observed behavior. Our work is then particularly relevant in the context of inverse optimal control [5]. A control problem involves designing the optimal control law, or state-to-action policy, to minimize a cost-function of a system, determined by task goals and energetic costs [4]. Exact solutions to the control problem can only be derived under linear dynamics, additive Gaussian noise, and a quadratic cost function, exploiting the independence between estimation and control [4]. However, when considering a realistic noise-model of the sensory-motor system (including multiplicative noise at the feedback and motor output levels and internal noise in the estimation process), this independence breaks down, requiring additional assumptions and approximations to derive optimal control laws [4]. In this work, we introduce two algorithms that outperform, in terms of cost minimization, state-of-the-art solutions for stochastic control problems in the presence of internal noise. We provide both heuristic and mathematical explanations for this improved performance, offering a practical application for sensory-motor control. These developments will allow stochastic control theory to be applied a to broader range of problems in systems neuroscience.

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