Random periodic non-uniform sampling sets for shift-invariant spaces

Diana Carbajal
University of Vienna

The sampling problem involves finding conditions under which a signal can be reconstructed from its samples taken on a discrete subset of its domain. The most classical assumption about the sampled signal is that it belongs to a Paley-Wiener space (the space of signals whose Fourier transform is supported on a given compact set). In multidimensional settings, finding a stable sampling set for this type of signal remains a very challenging task.

One prolific line of research considers the Paley-Wiener spaces whose spectrum has the property of multi-tiling the space along lattice translations. These signals admit a stable sampling set in the form of a finite union of translations of a lattice –a periodic non-uniform set– that meets Landau's density benchmark. However, in practice, explicitly constructing such sets requires a substantial level of effort. Namely, the practitioner would need to choose the translation points so as to avoid a certain algebraic variety of exceptional sets. Moreover, no simple a priori stability bounds are provided.

In this presentation, we discuss a probabilistic approach to this problem, while extending it to a broader signal model of shift-invariant spaces that includes the case of Paley-Wiener spaces with a multi-tiling spectrum. We show that by slightly exceeding Landau's density benchmark, it is possible to obtain non-uniform periodic sampling sets with overwhelming probability. The random sampling strategy not only provides a simple alternative with respect to the deterministic results but is also accompanied with explicit and possibly very favorable stability margins.

This is a joint work with Jorge Antezana (Autonomous University of Madrid) and José Luis Romero (University of Vienna).

- J. Antezana, D. Carbajal, J.L. Romero. *Random periodic sampling patterns for shift-invariant spaces*, at **IEEE Transactions on Information Theory**, (2023). `https://doi.org/10.1109/TIT.2023.3320289`