

Title:

Recent developments on Besicovitch's $\frac{1}{2}$ problem

Abstract:

Besicovitch's $\frac{1}{2}$ problem investigates the smallest threshold σ guaranteeing rectifiability for a set with Hausdorff 1-dimensional finite measure when the lower density of the set is larger than σ almost everywhere. Besicovitch conjectured that $\sigma = \frac{1}{2}$ (hence the name of the problem) and proved $\sigma \leq \frac{3}{4}$, then Preiss and Tišer improved the bound to $\sigma \leq \frac{2+\sqrt{46}}{12} \sim 0.73186\dots$. In a recent work in collaboration with C. De Lellis, F. Glaudo and D. Vittone, we devise a strategy to improve the bound by means of a hierarchy of variational problems and we reach a proof that $\sigma \leq 0.7$. In this seminar, I will try to explain the fairly intuitive geometric idea behind this strategy and I will try to summarize both the computational obstacles and the intrinsic obstacles that are still in the way.