New mathematical laws to describe the spatial distribution of individuals?

Á. Corral (CRM, UAB, CSH Vienna) and M. García del Muro (UB)

Using demographic data of high spatial resolution for a region in the south of Europe, we study the population over fixed-size (small) spatial cells.

We find that, counterintuitively, the distribution of the number of inhabitants per cell increases its variability when the size of the cells is increased. Nevertheless, the shape of the distributions is kept constant, which allows us to introduce a scaling law, analogous to finite-size scaling. This scaling indicates (empirically) the existence of a "new central limit theorem" (CLT), in which the resulting stable distribution is not normal (nor Lévy-stable) but seems to be well fitted by a gamma distribution. The reason for this is the existence of spatial correlations, which lead to the invalidity of the classical CLTs but to the validity of the "new CLT".

The scaling of the moments of the distribution is found to be related with the multifractal properties of the spatial pattern formed by the population. The agreement between theory and empirical data is satisfactory, yielding that only two exponents are necessary to describe the human-population pattern: $d_f=1.29$ and $\tau_2=1.69$.

We expect that our results could be extended from humans to other animals.

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