

WEIGHTED POINCARÉ-SOBOLEV INEQUALITIES VIA FRACTIONAL INTEGRATION

ALEJANDRO CLAROS

ABSTRACT. In this talk we will discuss weighted Poincaré-Sobolev type inequalities with an explicit analysis on the dependence on the A_p constants of the involved weights. We obtain inequalities of the form

$$\left(\frac{1}{w(B)} \int_B |f(x) - f_B|^q w(x) dx \right)^{\frac{1}{q}} \leq C_w r(B) \left(\frac{1}{w(B)} \int_B |\nabla f(x)|^p w(x) dx \right)^{\frac{1}{p}},$$

with quantitative estimates for the exponent q and the constant C_w , where $f_B = \frac{1}{|B|} \int_B f$ is the average of f over a ball B , $r(B)$ is the radius of B and $1 \leq p \leq q < \infty$. We prove this estimates as a consequence of the study of weighted local bounds for the fractional integral operator I_α , and the classical subrepresentation formula

$$|f(x) - f_B| \leq c_n I_1(|\nabla f| \chi_B)(x).$$

We also show that the dependence of C_w on the A_p constant is sharp. Also, we solve positively a conjecture of [PR19] related to the sharp exponent in the A_1 constant in the (p^*, p) Poincaré-Sobolev inequality with A_1 weights. Our method is very versatile, it allows us to prove Poincaré-Sobolev inequalities for high order derivatives and fractional Poincaré-Sobolev inequalities with the BBM extra gain factor $(1 - \delta)^{\frac{1}{p}}$.

REFERENCES

- [PR19] C. Pérez and E. Rela, *Degenerate Poincaré-Sobolev inequalities*, Trans. Amer. Math. Soc. **372** (2019), no. 9, 6087–6133, DOI 10.1090/tran/7775. MR4024515 ↑1

BCAM - BASQUE CENTER FOR APPLIED MATHEMATICS, BILBAO, SPAIN
Email address: aclaros@bcamath.org