Variational Formulations for Solving PDEs with Non-Smooth Solutions using Non-Linear Surrogates

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This talk intends to address the challenge of solving Partial Differential Equations (PDEs) with smooth or non-smooth solutions by formulating variational PDE formulations resulting in a soft-constrained optimization problem. The flexibility of the variational formulation enables us to use hybrid non-linear surrogates to approximate discontinuous shocks while solving forward or inverse PDE problems.

We first explore general concepts and tools necessary for solving PDEs under a variational formulation with general non-linear surrogates and boundary conditions. We then compare the numerical performance of Physics Informed Neural Networks (PINNs) as surrogates against Polynomial Surrogate Models (PSMs). Our goal is to open up the discussion regarding the class of problems that genuinely require the use of Neural Networks. Our findings indicate that PSMs outperform PINNs by several orders of magnitude in both accuracy and runtime.

Furthermore, we introduce a new method for approximating discontinuous functions using modified global spectral methods. We extend this method to solve PDEs with non-smooth solutions, providing an innovative solution to a highly challenging problem.