Complex dynamics of the quantum Cournot duopoly game with memory

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The study of game theory has been widely developed and applied in various fields, including economics, psychology, and biology. The use of quantum theory has been demonstrated as a way to improve game outcomes compared to classical games [1], thanks to the entanglement between players. Additionally, incorporating long-term memory has been shown to have a positive effect on the stability of a dynamical system [2]. In this context, in this work we have introduced memory into the dynamics of the quantum Cournot duopoly game [3].

We consider two cases: when both players are boundedly rational (homogeneous players) and when one player is boundedly rational and the other naive (heterogeneous players) comparing the results obtained in both cases. This second case it has been further studied in the literature.

We shown that neither memory nor the type of players (heterogeneous or homogeneous) produces variations in the fixed points comparing to the quantum, or even, classic Cournot duopoly game and, therefore, Nash equilibrium is preserved. However, their stability strongly depends on the entanglement (higher entanglement leads to more stability) and memory (more memory results in greater stability).

In addition to that, it is observed that the game with homogeneous players, which is not deeply studied previously in quantum games, can improve the local stability of the system versus the game with heterogeneous players, under certain conditions [4]. As it is expected, it is shown the role of memory as an effective mechanism of chaos control. This achievement is a remarkable economic advantage, since enables the system to reach the stability faster. In this work, these statements are proved analitically and supported widely with several numerical simulations, using different values of the memory factor

References

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