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Title: Small breathers of nonlinear Klein-Gordon equations via exponentially small homoclinic splitting

Abstract: Breathers are temporally periodic and spatially localized solutions of evolutionary PDEs. They are known to exist for integrable PDEs such as the sine-Gordon equation, but are believed to be rare for general nonlinear PDEs. When the spatial dimension is equal to one, exchanging the roles of time and space variables (in the so-called spatial dynamics framework), breathers can be interpreted as homoclinic solutions to steady solutions and thus arising from the intersections of the stable and unstable manifolds of the steady states. In this talk, we shall study small breathers of the nonlinear Klein-Gordon equation generated in an unfolding bifurcation as a pair of eigenvalues collide at the original when a parameter (temporal frequency) varies. Due to the presence of the oscillatory modes, generally the finite dimensional stable and unstable manifolds do not intersect in the infinite dimensional phase space, but with an exponentially small splitting (relative to the amplitude of the breather) in this singular perturbation problem of multiple time scales. This splitting leads to the transversal intersection of the center-stable and center-unstable manifolds which produces small amplitude generalized breathers with exponentially small tails. Due to the exponential small splitting, classical perturbative techniques cannot be applied. We will explain how to obtain an asymptotic formula for the distance between the stable and unstable manifold of the steady solutions. This is a joint work with O. Gomide, M. Guardia, and T. Seara.