

Study on the dynamics of coupled pendula

Abstract. Interacting pendula exhibit interesting and variegated dynamics, probably the most fascinating one being synchronization, as first observed in the 17th century by Huygens, mathematician and inventor of the first pendulum clocks. In this talk, I will present a realistic model consisting of an arbitrary number of pendula hanging on a viscoelastic string and discuss, in the regime of small oscillations, the spectrum of the system. Such analysis is not trivial, as it involves the interaction of finite-dimensional systems –the pendula– through a continuum medium –the string. As we shall see, two intrinsic properties of this system have a major impact on the distribution of the eigenvalues, namely the locally periodic arrangement of the pendula along the string and the dissipation of mechanical energy, modelled as internal damping within the string. As a result, the dynamics of the system presents multiple time-scales. In particular, we will be interested in the asymptotic behaviour of the system and in the understanding of the conditions under which it spontaneously evolves toward a state in which the pendula oscillate synchronously. Finally, we will discuss the nonlinear dynamics of the system invoking invariant manifold arguments.

This talk is based on a work in preparation jointly with F. Fassò and A. Ponno.