ADJOINT SYSTEMS VIA DIRAC STRUCTURES

BRIAN TRAN

ABSTRACT. In this talk, we begin by reviewing the notion of an adjoint system associated to an ordinary differential equation (ODE) on a manifold M. Given an ODE which is specified by a vector field on M, the adjoint system is the ODE on the cotangent bundle T^*M specified by the associated cotangent lifted vector field. Such adjoint systems arise naturally in the necessary conditions for optimality in control theory and are important in the stability analysis of control problems. Its use in stability analysis follows from the preservation of a quadratic invariant associated to symplecticity, since the adjoint system is Hamiltonian relative to the canonical symplectic structure on T^*M. Subsequently, we will review the notion of a Dirac structure and an implicit Hamiltonian system.

After developing the necessary background material on adjoint systems and Dirac structures, we extend the theory of adjoint systems for ODEs to the case of constrained ODEs. We construct such systems using the notion of a Dirac structure, which allows us to associate an implicit Hamiltonian system which generalizes the system associated to a cotangent lifted vector field. This generalization allows the theory of adjoint systems to be applied to control problems with constraints, which expands the range of applicability of such adjoint systems. After discussing properties of such constrained adjoint systems, we discuss several applications, including control theory and numerical partial differential equations (PDEs) and in particular, the semi-discretization of PDEs. This is joint work with Prof. Melvin Leok.

Time permitting, we will discuss a generalization of the concept of adjoint systems to the setting of PDEs. Such a generalization utilizes the notion of a multi-Dirac structure, which encodes the multisymplectic geometry of constrained Hamiltonian PDEs.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA, SAN DIEGO, 9500 GILMAN DRIVE, LA JOLLA, CA 92093-0112, USA.

E-mail address: b3tran@ucsd.edu.